IS 397-3 (2003): Method for Statistical Quality Control During Production, Part 3: Special Control Charts by Variables [MSD 3: Statistical Methods for Quality and Reliability]

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भारतीय मानक

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भाग 3 चरों के लिये विशेष नियंत्रण सारणी

(पहला पुनरीक्षण)

Indian Standard

METHODS FOR STATISTICAL QUALITY CONTROL DURING PRODUCTION

PART 3 SPECIAL CONTROL CHARTS BY VARIABLES

(First Revision)

ICS 03.120.30

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Price Group 7

Statistical Methods for Quality and Reliability Sectional Committee, MSD 3

FOREWORD

This Indian Standard (Part 3) (First Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

The efficacy of control charts in regulating production is quite well known. Part 1 of this standard covers traditional control charts for variables. This Part of the standard dealing with special control charts by variables has been prepared for use in those circumstances wherein the traditional control charts are not applicable, less efficient or more time consuming.

Since the basic philosophy for the use of control charts in manufacturing operations remains unaltered irrespective of the type of chart used, this Part should be read along with Part 1 for obtaining an integrated approach to the theory and practice of control charts. Part 1 of the standard is therefore necessary adjunct to this standard since many of the basic principles in the construction of control charts and their interpretation explained in Part 1 have not been repeated.

This standard was originally published in 1980. In view of the experience gained with the use of this standard in course of years, it was felt necessary to revise this standard so as to make the concepts more up-to-date. Following changes have been made in this revision:

a) Many editorial mistakes have been corrected, and
b) At certain places the concepts have been made clearer.

In addition to this Part, IS 397 has the following four parts:

IS No.	*Title*
397	Methods for statistical quality control during production:
(Part 0) : 2003	Guidelines for selection of control charts (*first revision*)
(Part 1) : 2003	Control charts for variables (*second revision*)
(Part 2) : 2003	Control charts for attributes (*third revision*)
(Part 4) : 2003	Special control charts by attributes (*first revision*)

The composition of the Committee responsible for the formulation of this standard is given in Annex B.

Indian Standard

METHODS FOR STATISTICAL QUALITY CONTROL DURING PRODUCTION

PART 3 SPECIAL CONTROL CHARTS BY VARIABLES

(First Revision)

1 SCOPE

This standard (Part 3) describes the following control charts with examples:

a) Group control charts,
b) Sloping control charts,
c) Moving averages and moving range charts,
d) Control charts for extreme values,
e) Control charts for coefficient of variation, and
f) Cumulative sum control chart.

2 REFERENCES

The following standards contain provisions, which through reference in this text constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

IS No.	*Title*
397 (Part 1) : 2003	Methods for statistical quality control during production: Part 1 Control charts for variables (*second revision*)
7920	Statistical vocabulary and symbols:
(Part 1) : 1994	Probability and general statistical terms (*second revision*)
(Part 2) : 1994	Statistical quality control (*second revision*)

3 TERMINOLOGY

For the purpose of this standard the definitions given in IS 7920 (Part 1) and IS 7920 (Part 2) shall apply.

4 GROUP CONTROL CHARTS

4.1 In industrial production it frequently so happens that the data presented for the purpose of controlling the quality comes from a number of sources, say from multi-spindle machine with same standard output, or several workers or several machines. In such cases unless proper steps are taken in choosing the sample, the quality engineer is hard put to single out the trouble-yielding source when control chart shows lack of control. One obvious way is to maintain a separate chart for each possible source, which is rather uneconomical and time-consuming. Group control chart, first devised with a view to controlling the dimensions on multiple-spindle automatics, which has wide applicability is a challenging answer to the problem.

4.2 The group control charts are valid only when there are enough reasons to presume that the means of each source of data as also the variability of each source are uniform. Instead of maintaining a pair of mean and range charts for each possible source like machine or worker, only one pair of mean and range charts are maintained. In the mean chart, the highest and lowest average values are plotted along with suitable source identifying indications (such as, serial number of machines/workers) and the largest range is plotted on the range chart. In the mean chart, the highest values are connected by a line, so also, the lowest values in order to avoid confusion. The underlying idea is that, if corresponding to a particular sample the highest value is below the upper control limit (*UCL*), the others are necessarily so and similarly if the lowest value is above the lower control limit (*LCL*), others are necessarily so. The identifying number attached to highest value that is beyond the *UCL* or to a lowest value that is below the *LCL* at once detects the trouble-yielding source.

4.3 For drawing group control chart for means, the various lines are obtained as follows:

$$CL = \bar{\bar{X}} = \Sigma \bar{\bar{X}}_i / k$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R}, \text{ where } \bar{R} = \sum_{i=1}^{k} R_i / k$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

where k is the total number of sources and $\bar{X}_i$ and R_i are the average and range corresponding to the ith source. $\bar{\bar{X}}$ and $\bar{R}$ are the averages of the homo-genized averages and ranges respectively.

Similarly for group control chart for range,

$$CL = \bar{R}$$

$$UCL = D_4 \bar{R}$$

$$LCL = D_3 \bar{R}$$

The values of the factors A_2, D_3, D_4 are given in Annex A for various sample sizes [*see* IS 397 (Part1)].

4.4 An advantageous use of group control charts requires that there should be several subgroup sources that yield approximately equal number of sub-groups at approximately the same rate, such as, different spindles on one automatic machine, several identical machines, or several operators each doing the same operation. There should be no difference between the averages or dispersions of various subgroups which cannot be corrected. For example, if there are ten machines on the same job but two of the machines have better process capability, then the group control chart cannot be applied for all the ten machines. The two machines, which give better process capability, should be treated separately.

4.4.1 The advantages of using a group control chart are:

a) It involves less work in plotting;
b) A compact presentation of all information on a single chart makes the interpretation easier; and
c) It is easier to find out whether a particular source is giving consistently high or low values on average or range chart. If there is no real difference among the sources, the numbers corresponding to the various sources should occur on the charts almost equally in the long run.

4.5 Example

Table 1 gives two measurements of the diameters of two pieces produced on each of six spindles of an automatic screw machine. The values given are in units of 0.001 mm in excess of 12 mm.

4.5.1 The highest and lowest average values are indicated in col 6 of Table 1 as *H* and *L* respectively. The highest ranges are also indicated by *H* in col 7 of Table 1.

4.5.2 *Control Limits for Range Chart*

$$CL = \bar{R} = 35/36 = 0.97$$
$$UCL = D_4\,\bar{R} = 3.267 \times 0.97 = 3.17$$
$$LCL = D_3\,\bar{R} = 0$$

Since all range values are less than $D_4\,\bar{R}$, the ranges are homogeneous.

4.5.3 *Control Limits for Average Chart*

$$CL = \bar{\bar{X}} = 195.5/36 = 5.43$$
$$UCL = \bar{\bar{X}} + A_2\,\bar{R} = 5.43 + 1.88 \times 0.97 = 7.25$$
$$LCL = \bar{\bar{X}} - A_2\,\bar{R} = 3.61$$

4.5.4 The average chart and range chart are plotted in Fig.1. In Fig. 1, the average chart, the highest and lowest average values along with suitable source identifying indications (spindle number) are plotted. Similarly in the range chart, the largest range along with suitable source identifying indications of spindle number is plotted.

5 SLOPING CONTROL CHARTS

5.1 In certain industries, the process level changes systematically during the course of production. The typical examples are that of tool wear in machine shop, changes due to fall in pressure as a tank empties, the effect of slowing down in reaction as a batch becomes weaker, etc. In the machine shop, it is desirable to sharpen the tools before the production of too many non-conforming items. On the other hand, it may not be advisable to unduly interrupt production for replacing or resharpening the tools. The problem is to economically minimize the total of the two costs – the cost of producing non-conforming products and the cost of replacing or resharpening tools.

5.2 In such cases ordinary $\bar{X} - R$ charts are not useful to control the process because the variation in the process is not only due to chance causes alone but also due to assignable causes such as tool wear. In this situation sloping control chart is found to be useful.

5.3 In sloping control chart samples are collected in such a manner that the:

a) production between two successive samples must be more or less constant; and
b) individual items of a sample must be consecutive items from the production so that the trend will have minimum effect on the range of the sample.

5.4 The average of the *k*th sample ($\bar{X}_k$) may be expressed as $\bar{X}_k = a + bk$ where *a* and *b* are constants. To set the control charts, the ranges are homogenized in the beginning and let $\bar{R}$ be the average of homogenized ranges.

The central line of the average chart is $a + bk$ where:

$$b = \sum_{k=1}^{n}\left(\bar{X}_k - \bar{\bar{X}}\right)\left(k - \bar{k}\right) / \sum\left(k - \bar{k}\right)^2,$$

where $\bar{\bar{X}}$ is the grand average.

$$= 6\sum_{k=1}^{n} 2\,\bar{X}_k\left(k - \bar{k}\right) / n(n+1)(n-1), \text{ and}$$

$$a = \bar{\bar{X}} - b\,\bar{k}$$

The control limits are:

For Average Chart:

$$CL = a + b\,k$$
$$UCL = a + b\,k + A_2\bar{R}$$
$$LCL = a + b\,k - A_2\bar{R}$$

Table 1 Diameter Measurements (Microns in Excess of 12 mm)
(*Clause* 4.5)

Sl No.	Sample No.	Spindle No.	Diameter		Average	Range	Remarks
			Piece 1	Piece 2	$\bar{X}$	R	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		1	6	7	6.5 *H*	1	
		2	4	6	5.0	2 *H*	
i)	1	3	6	4	5.0	2 *H*	
		4	5	4	4.5 *L*	1	
		5	6	5	5.5	1	
		6	4	5	4.5 *L*	1	
		1	6	6	6.0 *H*	0	
		2	6	6	6.0 *H*	0	
ii)	2	3	5	6	5.5	1	
		4	5	5	5.0 *L*	0	
		5	5	6	5.5	1	
		6	7	5	6.0 *H*	2 *H*	
		1	5	6	5.5	1 *H*	
		2	6	6	6.0 *H*	0	
iii)	3	3	5	5	5.0 *L*	0	
		4	6	5	5.5	1 *H*	
		5	5	5	5.0 *L*	0	
		6	6	6	6.0 *H*	0	
		1	5	6	5.5	1	
		2	6	5	5.5	1	
iv)	4	3	5	5	5.0	0	
		4	4	4	4.0 *L*	0	
		5	6	4	5.0	2 *H*	
		6	5	7	6.0 *H*	2 *H*	
		1	5	6	5.5	1	
		2	5	4	4.5 *L*	1	
v)	5	3	6	5	5.5	1	
		4	7	4	5.5	3 *H*	
		5	7	6	6.5 *H*	1	
		6	5	7	6.0	2	
		1	5	5	5.0 *L*	0	
		2	6	5	5.5	1	
vi)	6	3	4	7	5.5	3 *H*	
		4	7	6	6.5 *H*	1	
		5	5	5	5.0 *L*	0	
		6	6	5	5.5	1	
			Total		195.5	35	

For Range Chart:

$CL = \bar{R}$

$UCL = D_4\,\bar{R}$

$LCL = D_3\,\bar{R}$

where the values of A_2, D_3 and D_4 are given in Annex A for various sample sizes.

5.5 Example

Suppose there are 20 (equal to n) samples of 'Starter' machined with new tools, collected at regular (approximately) intervals of production and recorded in order of production. Each sample is of size 5. Table 2 gives the average and range of 'Head thickness' of the 20 samples in col 3 and 4 respectively. The control limits for the sloping control chart are calculated as follows:

$\bar{R} = 0.33/20 = 0.016\,5$

$D_4\bar{R} = 2.115 \times 0.016\,5 = 0.035$

All the range values are less than $D_4\,\bar{R}$. Hence they are all homogeneous.

$$\bar{\bar{X}} = \left(\sum_{k=1}^{20} \bar{X}_k\right) / 20 = 39.504 / 20 = 1.975\,2$$

$$\bar{k} = (n+1)/2 = 10.5$$

$$\sum_{k=1}^{20} 2\bar{X}_k\left(k-\bar{k}\right) = 1.896$$

$$b = 6 \times \sum_{k=1}^{20} 2\bar{X}_k\left(k-\bar{k}\right)/n\,(n+1)\,(n-1) =$$

$$(6 \times 1.896)/(20 \times 21 \times 19) = 0.001\,4$$

$$a = \bar{\bar{X}} - b\,\bar{k} = 1.975 - 0.001\,4 \times 10.5 = 1.960\,5$$

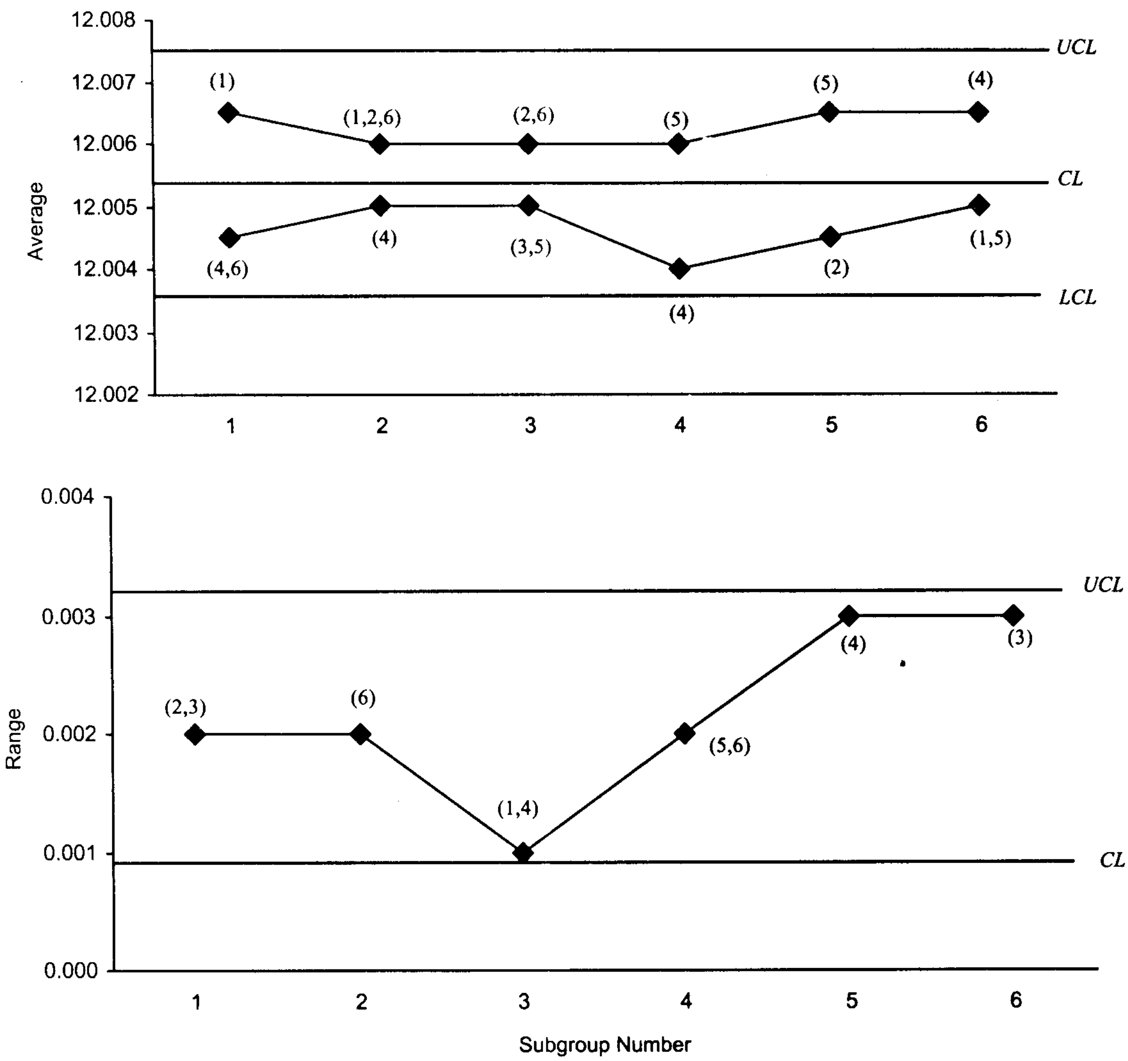

FIG. 1 GROUP CONTROL CHART

$$A_2\overline{R} = 0.577 \times 0.016\ 5 = 0.009\ 5$$

5.5.1 *For Average Chart:*

$$CL = 1.960\ 5 + 0.001\ 4\ k$$
$$UCL = 1.970\ 0 + 0.001\ 4\ k$$
$$LCL = 1.951\ 0 - 0.001\ 4\ k$$

For Range Chart:

$$CL = 0.0165$$
$$UCL = 0.035$$
$$LCL = D_3\overline{R} = 0$$

To use the control chart during production, the sample shall be drawn at equal intervals of production and $\overline{X}_k$ and R_k will be plotted in the chart against the corresponding k of the X axis. The criteria for out of control situation are the same as that in ordinary $\overline{X} - R$ chart.

5.5.2 The data given in Table 2 have been plotted on sloping control chart (for average) and range chart in Fig. 2.

6 MOVING AVERAGES AND MOVING RANGE CHARTS

6.1 In certain cases of industrial production it takes considerable time to produce a new item, so it is inconvenient to sample frequently. But in the mean time process average or dispersion may shift and this may incur some appreciable loss to the producer.

6.1.1 To obviate this, use of moving averages and moving ranges instead of ordinary averages and ranges has been suggested. Moving averages of k items are obtained as follows. Initially, the first k values are averaged. Then in the second step the first value is dropped in favour of the $(k+1)$th value and an average obtained. Next, the second value is dropped and the $(k+2)$th value included and these values are averaged, and so on. In a similar manner moving ranges are obtained.

Table 2 Computations for the Sloping Control Chart

(*Clauses* 5.5 *and* 5.5.2)

Sl No.	Sample No. k	Average $\bar{X}_k$ mm	Range R_k mm	$2(k-\bar{k})$	$2(k-\bar{k})\bar{X}_k$	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i)	1	1.962	0.03	–19	–37.278	
ii)	2	1.964	0.00	–17	–33.388	
iii)	3	1.960	0.03	–15	–29.400	
iv)	4	1.966	0.02	–13	–25.558	
v)	5	1.968	0.03	–11	–21.648	
vi)	6	1.968	0.02	–9	–17.712	
vii)	7	1.970	0.01	–7	–13.790	
viii)	8	1.974	0.02	–5	–9.870	
ix)	9	1.972	0.00	–3	–5.916	
x)	10	1.976	0.01	–1	–1.976	
xi)	11	1.976	0.01	1	1.976	
xii)	12	1.980	0.01	3	5.940	
xiii)	13	1.978	0.02	5	9.890	
xiv)	14	1.982	0.03	7	13.874	
xv)	15	1.984	0.01	9	17.856	
xvi)	16	1.980	0.03	11	21.780	
xvii)	17	1.984	0.00	13	25.792	
xviii)	18	1.986	0.02	15	29.790	
xix)	19	1.986	0.03	17	33.762	
xx)	20	1.988	0.00	19	37.772	
	Total	39.504	0.33	0	1.896	

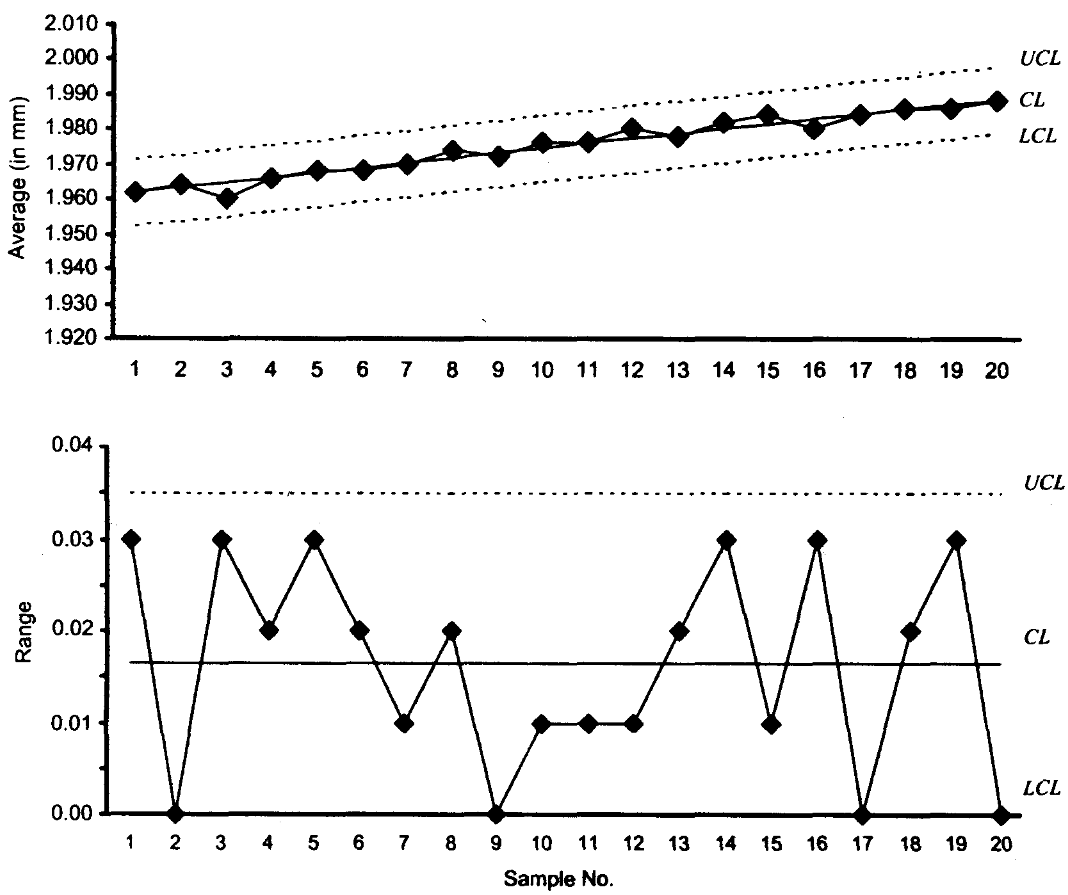

FIG. 2 AVERAGE (SLOPING) AND RANGE CHART

6.2 Suppose there are 15 observations. In the table, which follows the 5-item moving averages and moving ranges, are calculated. It may be noted that for 15 observations, the number of 5 items moving averages and ranges would be 15 + 1 – 5 = 11.

Sl No.	*Observation*	*Moving Average*	*Moving Range*	*Remarks*
(1)	(2)	(3)	(4)	(5)
1	8	–	–	–
2	7	–	–	–
3	8	(8+7+8+9+8)/5 = 8.0	9 – 7 = 2	–
4	9	(7+8+9+8+5)/5 = 7.4	9 – 5 = 4	–
5	8	(8+9+8+5+6)/5 = 7.2	9 – 5 = 4	–
6	5	(9+8+5+6+7)/5 = 7.0	9 – 5 = 4	–
7	6	(8+5+6+7+5)/5 = 6.2	8 – 5 = 3	–
8	7	(5+6+7+5+9)/5 = 6.4	9 – 5 = 4	–
9	5	(6+7+5+9+9)/5 =7.2	9 – 5 = 4	–
10	9	(7+5+9+9+8)/5 = 7.6	9 – 5 = 4	–
11	9	(5+9+9+8+8)/5 = 7.8	9 – 5 = 4	–
12	8	(9+9+8+8+7)/5 = 8.2	9 – 7 = 2	–
13	8	(9+8+8+7+6)/5 = 7.6	9 – 6 = 3	–
14	7	–	–	–
15	6	–	–	–

6.3 Control charts are set up in exactly the same manner as in the case of ordinary averages and ranges. Usually moving average and moving range control charts are employed for the purpose of current control. The overall mean $\overline{\overline{X}}$ and the average range $\overline{R}$ are obtained from supplied past data and these are utilized to set up control limits as follows:

Moving Range Chart

$$CL = \overline{R}$$
$$UCL = D_4\overline{R}$$
$$LCL = D_3\overline{R}$$

Moving Average Chart

$$CL = \overline{\overline{X}}$$
$$UCL = \overline{\overline{X}} + A_2\overline{R}$$
$$LCL = \overline{\overline{X}} - A_2\overline{R}$$

where A_2, D_3 and D_4 have been tabulated in Annex A for various sample sizes.

6.4 When the control charts have been set up, the operation proceeds with the plotting of moving average and moving ranges in chronological order. In a k item moving average $(k - 1)$ values are common between any two successive averages (or ranges), $(k - 2)$ values are common between any two alternate values and so on. So unlike the case of ordinary control charts, here successive points are not independent. In fact, in a k-item moving average series only values (average of range) which are $(k + 1)$ intervals apart are independent. Hence, in moving control charts, runs on either side of the central line, do not tell the same story as does an ordinary control chart. However, a point out of control limits here has the same significance as in an ordinary chart.

6.5 The advantage of a control chart for moving average or moving range is that it gives a warning signal earlier. It is not necessary to wait until an entire new sample is accumulated. This may be important if the product is either expensive or the rate of output is small.

6.5.1 Since the probability of obtaining a run of any kind is much larger with control chart for moving average or moving range as compared to the orthodox type of charts, the traditional interpretation of runs is not valid for these type of charts.

7 CONTROL CHARTS FOR EXTREME VALUES

7.1 It may at times be desirable to use a control chart for largest and smallest values or the high-low control chart, as it is popularly called, in place of the

conventional charts for the averages and ranges. These type of charts are extremely simple since no calculations are needed for plotting the points. Besides, only one chart needs to be maintained in this case in place of the two conventional charts since information concerning both central tendency and dispersion are provided on the single chart.

7.2 Constants for Determining Limits

Let L and S denote the largest and smallest values respectively in a sample of n pieces, and let $\bar{L}$ and $\bar{S}$ be the average of these values for k samples. Then $(\bar{L} + \bar{S})/2$ and $(\bar{L} - \bar{S})/d_2$ and are unbiased estimates of population mean and standard deviation respectively, in the case of a random sample from a normal population.

Let $(\bar{L} + \bar{S})/2 = M = \mu,$

$\bar{L} - \bar{S} = \bar{R} = d_2\,\sigma$

7.3 Mean and Standard Deviation not Known

When the values of the process average and dispersion are not known from the past data, they are estimated with the help of the initial data collected and the central lines and control limits are computed as follows:

$$CL = (\bar{L}+\bar{S})/2 = M$$
$$UCL = M + H_2\,\bar{R}$$
$$LCL = M - H_2\,\bar{R}$$

Where the values of H_2 are given in Annex A for sample sizes 2 to 6 and $\bar{R} = \bar{L} - \bar{S}$.

NOTE — Since the efficiency of M decreases rapidly with the increasing sample size, it may be better to substitute grand average $\bar{X}$ for M in computing both the central line and the control limits.

7.4 Mean and Standard Deviation Known

When control charts for extreme values are used to control a current process, and if the values of process average and dispersion are known as μ and σ respectively, then to plot both the extremes values on a single chart, the control limits and the central line are set up as:

$$CL = \mu$$
$$UCL = \mu + H\sigma$$
$$LCL = \mu - H\sigma$$

where the value of H for sample sizes 2 to 6 are as given in Annex A.

7.5 It has been found that under most condition, L and S chart is nearly as good as the $\bar{X}$ and R charts for detecting lack of control. Like in the case of individual chart, the specification limits may validly be drawn in the case of a control chart for extreme values.

7.6 Although an upper control limit for L and a lower control limit for S are usually drawn, it is possible to have an upper and a lower control limit for each L and S separately. The control limits for L and S are given by $\bar{L} \pm (H_2 - 1/2)\,\bar{R}$ and $\bar{S} \pm (H_2 - 1/2)\,\bar{R}$ respectively. In such a case, a shift in the process; is indicated if both L and S are above the respective upper control limits or below the lower control limits. On the other hand, if L is above the relevant upper control limit and S is below the relevant lower control limit, this is enough evidence to conclude the increase in process variability.

7.7 Example

7.7.1 Suppose there are 25(n) samples of 'bolts' machined on Turret lathe, collected at regular intervals of production and recorded in order of production. Each sample is of size 5. Table 3 gives the largest (L) and smallest (S) values of head diameter of 25 samples in col 3 and 4 respectively. The control limits for the extreme value chart are calculated as follows:

$$\bar{L} = \sum_{i=1}^{k} L_i / k = 99.61/25 = 3.984 \text{ and}$$

$$\bar{S} = \sum_{i=1}^{k} S_i / k = 98.98/25 = 3.959$$

Unbiased estimate of mean (M) $= (\bar{L} + \bar{S})/2 = 3.97$

Unbiased estimate of standard deviation

$$(\sigma) = \frac{\bar{R}}{d_2} = \frac{\bar{L} - \bar{S}}{d_2} = \frac{0.025}{2.326} = 0.011$$

7.7.2 Central line and control limits for the control chart for extreme values are:

$$CL = M = 3.97$$
$$UCL = M + H_2\,\bar{R} = 3.97 + 1.36 \times 0.025 = 4.00$$
$$LCL = M - H_2\,\bar{R} = 3.94$$

7.7.3 To use the control chart during production, the sample will be drawn at regular intervals of production and the largest value (L) and the smallest value (S) will be plotted in the chart against the sample number k on the X-axis. The criteria for out of control situation is same as that of ordinary $\bar{X} - R$ chart.

7.7.4 For the purpose of illustration, data of Table 3 have been plotted on a control chart for extreme values in Fig. 3.

8 CONTROL CHART FOR COEFFICIENT OF VARIATION

8.1 Coefficient of variation (V) may give a useful characterization of variability in cases where samples are drawn from populations with different means and standard deviations but with the same ratio of these two measures as is, for instance, the case with the strength of concrete. In fact, the mean compressive strength of cement mortar cubes increases with the curing time, and the standard deviation increases in

the same proportion, but the ratio $s / \bar{x}$ being practically constant. Other examples are provided by the crushing strength of bricks, the sliver thickness of jute-fibres at different preparing stages (carding and drawing, etc).

Table 3 Largest and Smallest Values of Head Diameter

(*Clauses* 7.7.1 *and* 7.7.4)

Sl No. (1)	Sample No. K (2)	Largest Value L mm (3)	Smallest Value S mm (4)	Remarks (5)
i)	1	4.00	3.96	
ii)	2	3.99	3.95	
iii)	3	3.99	3.97	
iv)	4	4.00	3.97	
v)	5	3.99	3.97	
vi)	6	4.00	3.97	
vii)	7	3.98	3.96	
viii)	8	3.99	3.98	
ix)	9	4.00	3.98	
x)	10	3.99	3.97	
xi)	11	4.00	3.98	
xii)	12	4.01	3.98	
xiii)	13	3.98	3.97	
xiv)	14	4.00	3.98	
xv)	15	3.98	3.97	
xvi)	16	3.96	3.95	
xvii)	17	3.96	3.94	
xviii)	18	3.96	3.94	
xix)	19	3.98	3.93	
xx)	20	3.98	3.93	
xxi)	21	3.97	3.94	
xxii)	22	3.97	3.95	
xxiii)	23	3.97	3.94	
xxiv)	24	3.97	3.95	
xxv)	25	3.97	3.95	
	Total	99.61	98.98	

8.2 The relative variability may then be controlled by computing the sample coefficient of variation and plotting these on a control chart. High values of this hybrid statistic will result from high variability of quality or poor mean quality both of which are taken to be indicative of unsatisfactory products. Correspondingly low values of the coefficient of variation are considered favourable. Therefore, in control charts for coefficient of variation, it is only necessary to red-flag the advent of a manufacturing trouble causing a too high value of the sample coefficient of variation. Thus, one is concerned with the upper limit mainly and this limit should be so set that a sample value larger than this comes from the specified population only with a probability less than the pre-assigned quantity.

8.3 For setting up a control chart for coefficient of variation, the various lines are obtained as follows:

$$CL = \bar{V} \qquad \left[\bar{V} = \left(\sum_{i=1}^{k} CV_i\right) / k\right]$$
$$UCL = B_4 \, \bar{V}$$
$$LCL = B_3 \, \bar{V}$$

where the factors B_3 and B_4 are given in Annex A for various sample sizes.

8.4 If the value of the process average (μ) and process standard deviation (σ) are known either from past experience or records, then the control charts for coefficient of variation are set up as follows:

$$CL = c_2 \, \sigma/\mu$$
$$UCL = B_2 \, \sigma/\mu$$
$$LCL = B_1 \, \sigma/\mu$$

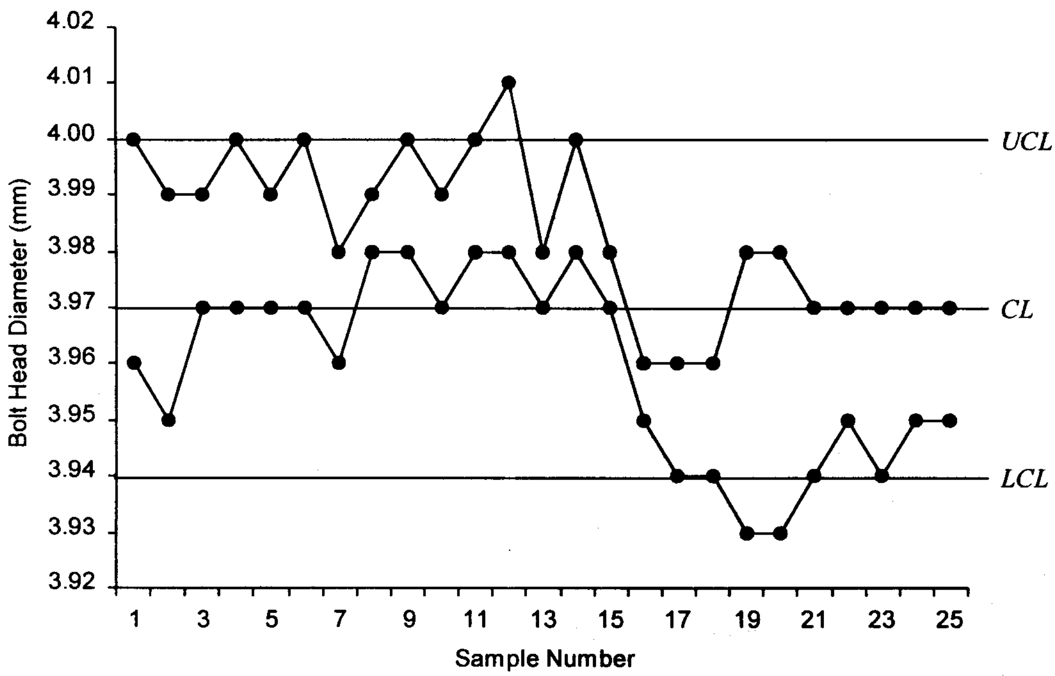

FIG. 3 CONTROL CHART FOR EXTREME VALUES

where the values of factors c_2, B_1 and B_2 have been tabulated in Annex A for various sample sizes.

8.5 Example

In the jute industry, uniformity of linear density of sliver is an important criterion which affects the subsequent operations of spinning and weaving. It is, therefore, desirable to control this property properly. With a view to installing a control chart for coefficient of variation weight of five 10 m lengths of sliver were daily collected at finisher card stage. The records of 25 days at a particular reference moisture regain are presented in Table 4 along with the average and *CV* percent. (The observations in a particular sample correspond to one machine only at a particular time.)

8.5.1 For the *CV* Control Chart:

$$CL = \bar{V} = 106.53/25 = 4.26$$
$$UCL = B_4\,\bar{V} = 2.089 \times 4.26 = 8.90$$
$$LCL = B_3\,\bar{V} = 0 \times 4.26 = 0$$

8.5.2 The data of Table 4 along with control limits have been plotted on a control chart for coefficient of variation in Fig. 4.

9 CUMULATIVE SUM CONTROL CHARTS

9.1 A standard control chart for controlling the mean of a normal population consists of a plot of observed sample means in sequence which are then compared against a set of fixed limits drawn parallel to the central line representing the 'aimed at mean'. Such control charts suffer from the disadvantage of the sample results being viewed individually. Consequently the traditional control charts are less sensitive for detecting moderate shifts in the mean value. One way of remedying the situation is through the increase of sample size which has many practical limitations. An alternative is to make use of the cumulative sum control charts (or cusum charts as it is popularly known) which are quite sensitive to detect small shifts in the process average. In cumulative sum control charts, the points plotted do not represent a single observation or statistic calculated from one sample only. Starting from a given point, all subsequent plots contain information from all the observations up to and including the plotted point. Thus the points on a cumulative sum control chart have coordinate (m, T_m), where m is the serial number of the sample and T_m represents the value of the statistic computed from all the m samples taken collectively.

Table 4 Weights of 10-Metre Lengths of Slivers

(*Clauses* 8.5 *and* 8.5.2)

Sl No.	Sample No.	Weights of 10-Metre Lengths (g) at 25 Percent Moisture					Average	*CV*	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
i)	1	751	681	708	748	704	718.4	3.76	
ii)	2	808	794	839	887	811	827.8	3.99	
iii)	3	760	731	774	771	765	760.2	2.02	
iv)	4	794	794	777	774	811	790.0	1.70	
v)	5	714	712	695	697	703	704.2	1.09	
vi)	6	735	735	760	705	764	739.8	2.87	
vii)	7	730	735	780	715	705	733.0	3.52	
viii)	8	735	820	700	765	790	762.0	5.48	
ix)	9	740	745	705	765	715	734.0	2.93	
x)	10	695	725	745	730	770	733.0	3.36	
xi)	11	645	640	685	660	657	657.4	2.38	
xii)	12	655	690	605	618	655	644.6	4.68	
xiii)	13	662	682	655	705	670	674.8	2.60	
xiv)	14	620	610	630	610	695	633.0	5.04	
xv)	15	760	710	740	690	745	729.0	3.48	
xvi)	16	632	703	688	655	740	683.6	5.50	
xvii)	17	720	700	681	777	792	734.0	5.90	
xviii)	18	600	612	697	775	780	692.8	11.09	
xix)	19	717	690	753	686	681	705.4	3.81	
xx)	20	795	822	707	725	774	764.5	5.61	
xxi)	21	605	715	764	655	660	679.8	8.04	
xxii)	22	810	740	825	733	796	780.8	4.79	
xxiii)	23	650	600	693	651	666	652.0	4.04	
xxiv)	24	700	767	720	783	710	736.0	4.46	
xxv)	25	665	640	700	653	704	672.4	3.79	
							Total	106.53	

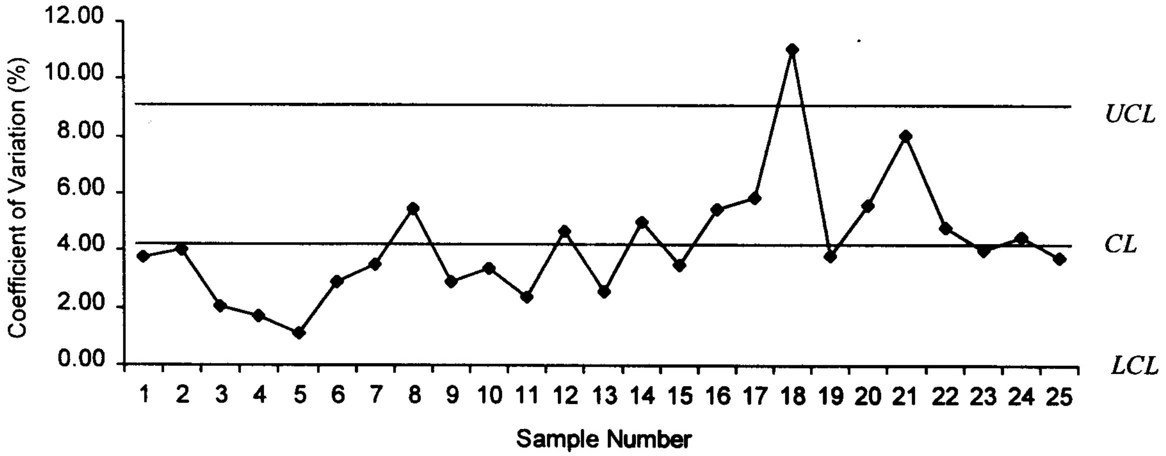

FIG. 4 CONTROL CHART FOR COEFFICIENT OF VARIATION

9.2 In a cumulative sum control chart for means, cumulative totals are plotted against the sample number. Denoting the mean of the *m*th sample by $\bar{x}_m$ and the target mean as μ, the *m*th point plotted has co-ordinates (m, T'_m).

$T'_m = [(\bar{x}_1 - \mu) +) (\bar{x}_2 - \mu) + ... + (\bar{x}_m - \mu)]/ \sigma_{\bar{x}}$

where $\sigma_{\bar{x}} = \sigma/\sqrt{x}$ if *n* is the size of each sample.

9.2.1 For the sake of simplicity, division by $\sigma_{\bar{x}}$ is avoided so that the *m*th point plotted has co-ordinates (m, T_m) where

$$T_m = \sum_{i=1}^{m} (\bar{x}_i - \mu)$$

9.2.2 The points are plotted on a graph with an ordinate scale specially adjusted (*see* Note under **9.6**).

9.3 The chart is interpreted by placing a *V*-shaped mask (*see* area in Fig. 5) over the chart, with the point *O* coinciding with the last point plotted on the chart and the line *OP* horizontal. Lack of control is diagnosed if any of the previously plotted points are covered by the mask, that is, if any points lie below the straight line *A'B'*, or above the straight line *A'B'*. A point below the former line indicates an increase in the process average, while a point above the latter line indicates a decrease.

9.4 The dimensions of the mask are defined by the angle (2θ) between *AB* and *A'B'* and the distance *d* from *O* to the vertex *P* of this angle. The quantities *d* and θ should be so determined that :

a) a lack of control will be indicated when the process is in control with a known small probability, and
b) a departure in process mean to a specified extent will be detected with a high known probability.

9.5 For the application of the cusum chart, it is essential to know the value of the following parameters before hand:

μ_0 = satisfactory process average of the characteristic to be controlled;

σ = standard deviation of the process when it is in a state of statistical control;

D = change in the mean (absolute) which is desired to be detected with fair certainty; and

n = sample size.

9.6 From the known values of the parameters, calculate *D/k* and $D\sqrt{n}/\sigma$ and enter Table 5 twice, once for value of *D/k* and again for $D\sqrt{n}/\sigma$ to obtain the corresponding values of θ and *d* respectively for the preparation of the mask.

NOTE — *k* is the number of units of the ordinate to one unit of the abscissa, which corresponds to the horizontal distance between successive plotted points. It is preferable to choose *k* such that it is approximately equal to $\sigma/\sqrt{n}$

9.7 Using the above values of θ and *d*, a mask (transparent) may be prepared. From the successive samples of size *n*, the sample mean shall then be calculated. From each of the sample mean, is deducted and the resultant deviation $(\bar{x}_i - \mu)$ shall be summed and plotted against the serial number of the sample. At each plotting, the mask shall be used as described in **9.3** to find out whether the process is in control or not.

9.8 If more than one point is covered by the mask at any instant, then the first point (from the left) covered by the mask indicates the time at which the shift in the process average started. If none of the points are covered, it is an indication that the process is in a state of statistical control.

9.9 Since the operation of the cusum chart assumes a constant standard deviation of the process, it is necessary to maintain a range chart along with the

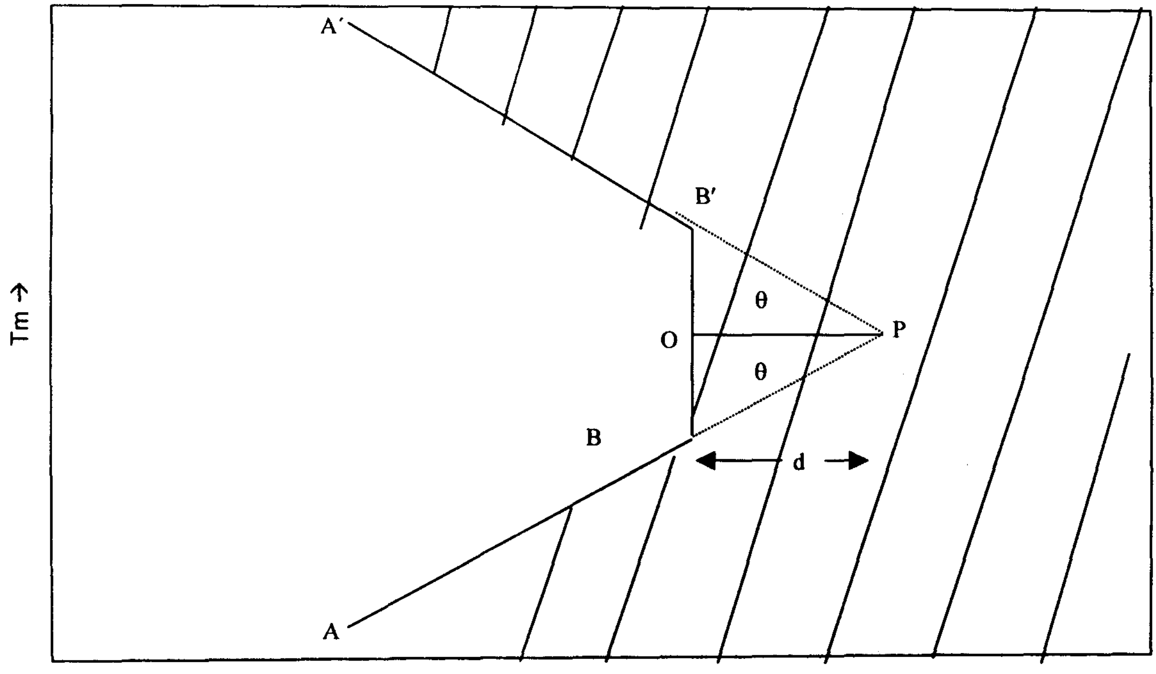

FIG. 5 MASK FOR CUMULATIVE SUM CONTROL CHART

Table 5 Calculation of θ and d
(*Clause* 9.6)

(D/k) or ($D\sqrt{n}/\sigma$)	θ	d	Remarks
(1)	(2)	(3)	(4)
0.1	2°52′	1 321.5	
0.2	5°43′	330.4	
0.3	8°32′	146.8	
0.4	11°19′	82.6	
0.5	14°02′	52.9	
0.6	16°42′	36.7	
0.7	19°17′	27.0	
0.8	21°48′	20.6	
0.9	24°14′	16.3	
1.0	26°34′	13.2	
1.1	28°49′	10.9	
1.2	30°58′	9.2	
1.3	33°01′	7.8	
1.4	35°00′	6.7	
1.5	36°52′	5.9	
1.6	38°40′	5.2	
1.7	40°22′	4.6	
1.8	41°59′	4.1	
1.9	43°32′	3.7	
2.0	45°00′	3.3	
2.1	46°24′	3.0	
2.2	47°44′	2.7	
2.3	48°59′	2.5	
2.4	50°12′	2.3	
2.5	51°20′	2.1	
2.6	52°26′	2.0	
2.7	53°28′	1.8	
2.8	54°28′	1.7	
2.9	55°24′	1.6	
3.0	56°19′	1.5	

cusum chart. Also, if the process standard deviation (σ) is not known, it can be estimated by $\bar{R}/d_2$ where $\bar{R}$ is the average range and d_2 is a factor to be obtained from Annex A for the different sample sizes.

9.10 Example

9.10.1 In a textile mill, it was decided to install a cumulative sum control chart for means for controlling the weight of drawing sliver. For the purpose, the weights (in g) of 5 m of drawing sliver for 15 samples each of size 5 are given in Table 6. The dimensions of V-mask are computed with the following information given:

μ	=	satisfactory process average to be attained	=	20 g
σ	=	standard deviation of process	=	0.4 g
D	=	shift in the mean which is desired to be detected with high probability	=	0.3 g
n	=	the size of sample	=	5

9.10.2 To calculate the dimensions of the mask, initially the mean of each sample (*see* col 8 of Table 6) is obtained and the deviations of the means from the process average (*see* col 9 of Table 6) and cumulative sum of deviations (T_m) (*see* col 10 of Table 6) are also calculated.

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.4/\sqrt{5} = 0.18$$

k = the scale factor of ordinate (obtained by rounding off the value of $\sigma_{\bar{x}}$) = 0.2

D/k = 0.3/0.2=1.5

$D\sqrt{n}/\sigma$ = $0.3 \times \sqrt{5}/0.4 = 1.7$

From Table 5, the dimensions of mask are obtained as:

d = 4.6 (corresponding to value 1.7 of $D\sqrt{n}/\sigma$)

θ = 36°52′ (corresponding to value 1.5 of D/k).

9.10.3 For the purpose of illustration, T_m values (*see* col 10 of Table 6), are plotted against the various sample numbers in Fig. 6. At each plotting, the suitably prepared *V*-mask (with θ = 36°52′ and d = 4.6) is superimposed on the graph with the point *O* of the mask coinciding with the plotted point and *OP* parallel to the horizontal axis. It may be seen that at the last point (15) plotted, the *V*-mask superimposition covers the point corresponding to eleventh subgroup, indicating that there has been a significant upward shift in the process average at that time.

Table 6 Weights of 5 Metres of Drawing Slivers (in Grams)

(*Clause* 9.10.1, 9.10.2 *and* 9.10.3)

Sample M	Individual Weighings g 1	2	3	4	5	Total	Mean $\bar{x}_m$	Deviation from Target $\bar{x}_m - \mu$	$T_m = \Sigma(\bar{x}_m - \mu)$	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	19.99	20.23	19.50	19.94	19.63	99.29	19.86	–0.14	–0.14	
2	20.22	19.84	20.18	20.23	19.49	99.96	19.99	–0.01	–0.15	
3	19.76	20.41	20.08	20.10	19.97	100.32	20.06	0.06	–0.09	
4	20.17	20.11	19.66	19.91	20.82	100.67	20.13	0.13	0.04	
5	19.72	19.95	20.03	20.03	20.43	100.16	20.03	0.03	0.07	
6	19.83	20.24	19.07	19.16	20.09	98.39	19.68	–0.32	–0.25	
7	20.33	19.87	19.44	19.98	20.04	99.66	19.93	–0.07	–0.32	
8	19.91	19.42	19.96	19.63	19.81	98.73	19.75	–0.25	–0.57	
9	19.83	20.46	19.49	19.42	19.91	99.11	19.82	–0.18	–0.75	
10	20.02	20.05	20.02	20.01	20.17	100.27	20.05	0.05	–0.70	
11	20.13	20.06	19.60	19.62	20.19	99.60	19.92	–0.08	–0.78	
12	20.82	20.58	20.21	20.50	20.30	102.41	20.48	0.48	–0.30	
13	20.34	20.04	20.66	20.21	20.20	101.45	20.29	0.29	–0.01	
14	20.04	20.31	20.19	20.46	20.05	101.05	20.21	0.21	–0.20	
15	20.16	20.13	20.23	21.07	19.99	101.58	20.32	0.32	0.52	

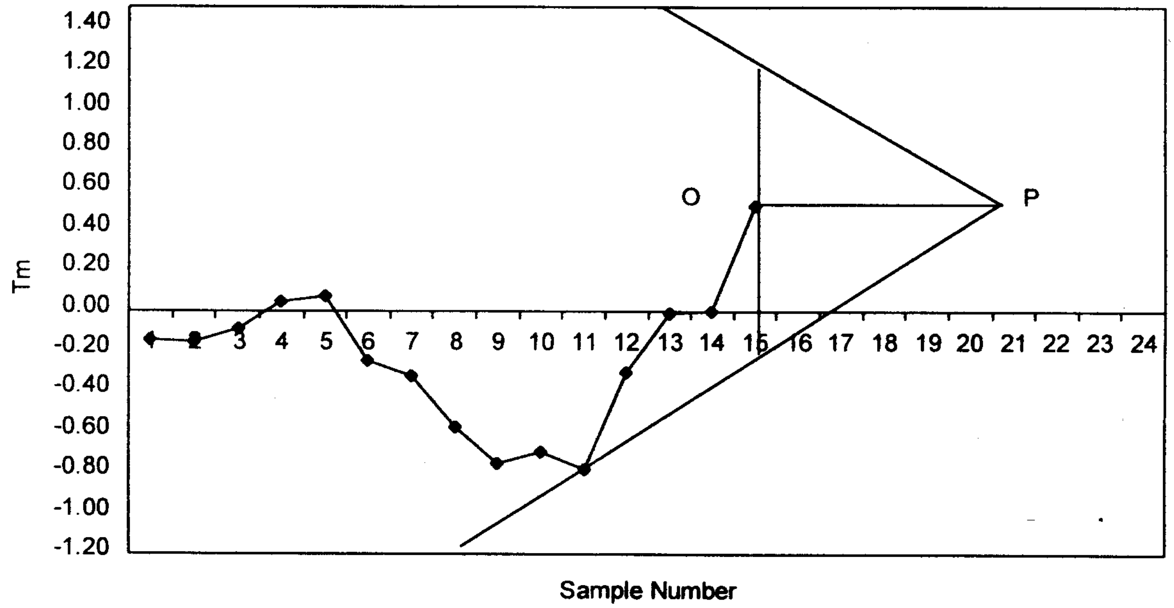

FIG. 6 CUMULATIVE SUM CONTROL CHART FOR MEAN

ANNEX A

(*Clauses* 4.3, 5.4, 6.3, 7.3, 7.4, 8.3, 8.4 *and* 9.9)

FACTORS FOR COMPUTING CONTROL LIMITS

	Using Standard Values of μ and σ								*Using (s)*		*Using R*			
	Extreme Value Chart	*CV Chart*			*Range Chart*				*CV Chart*		*Extreme Value Chart*	*Average Chart*	*Range Chart*	
CL	μ	$c\sigma/\mu$			$d_2\sigma$				$\bar{V}$		$M=\frac{\bar{L}+\bar{S}}{2}$	$\bar{\bar{X}}$	$\bar{R}$	
LCL	$\mu + H\sigma$	$B_1\,\sigma/\mu D_1\sigma$			$D_1\sigma$				$B_3\bar{V}$		$M+H_2\bar{R}$	$\bar{\bar{X}}+A_2\bar{R}$	$D_3\bar{R}$	
UCL	$\mu + H\sigma$	$B_2\,\sigma/\mu D_2\sigma$			$D_2\sigma$				$B_4\bar{V}$		$M+H_2\bar{R}$	$\bar{\bar{X}}+A_2\bar{R}$	$D_4\bar{R}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
No. of observations in the sample	H	c_2	B_1	B_2	d_2	D_1	D_2	B_3	B_4	H_2	A_2	D_3	D_4	
2	3.03	0.564 2	0	1.843	1.128	0	3.686	0	3.267	2.72	1.880	0	3.267	
3	3.09	0.723 6	0	1.858	1.693	0	4.358	0	2.568	1.82	1.023	0	2.575	
4	3.15	0.797 9	0	1.808	2.059	0	4.698	0	2.266	1.53	0.729	0	2.282	
5	3.17	0.840 7	0	1.756	2.326	0	4.918	0	2.089	1.36	0.577	0	2.115	
6	3.21	0.868 6	0.026	1.711	2.534	0	5.078	0.030	1.970	1.27	0.483	0	2.004	
7		0.888 2	0.105	1.672	2.704	0.205	5.203	0.118	1.882	–	–	–	–	
8		0.902 7	0.167	1.638	2.847	0.387	5.307	0.185	1.815	–	–	–	–	
9		0.913 9	0.219	1.609	2.970	0.546	5.394	0.239	1.761	–	–	–	–	
10		0.922 7	0.262	1.584	3.078	0.687	5.469	0.284	1.716	–	–	–	–	
11		0.930 0	0.299	1.561	3.173	0.812	5.534	0.321	1.679	–	–	–	–	
12		0.935 9	0.331	1.541	3.258	0.924	5.592	0.354	1.646	–	–	–	–	
13		0.941 0	0.359	1.523	3.336	1.026	5.646	0.382	1.618	–	–	–	–	
14		0.945 3	0.384	1.507	3.407	1.121	5.693	0.406	1.594	–	–	–	–	
15		0.949 0	0.406	1.492	3.472	1.207	5.737	0.428	1.572	–	–	–	–	
16		0.952 3	0.427	1.478	3.532	1.285	5.779	0.448	1.552	–	–	–	–	
17		0.955 1	0.445	1.465	3.588	1.359	5.817	0.466	1.534	–	–	–	–	
18		0.957 6	0.461	1.454	3.640	1.426	5.854	0.482	1.518	–	–	–	–	
19		0.959 9	0.477	1.443	3.689	1.490	5.888	0.497	1.503	–	–	–	–	
20		0.961 9	0.491	1.433	3.735	1.548	5.922	0.510	1.490	–	–	–	–	
21		0.963 8	0.504	1.424	3.778	1.606	5.950	0.523	1.477	–	–	–	–	
22		0.965 5	0.516	1.415	3.819	1.659	5.979	0.534	1.466	–	–	–	–	
23		0.967 0	0.527	1.407	3.858	1.710	6.006	0.545	1.455	–	–	–	–	
24		0.968 4	0.538	1.399	3.895	1.759	6.031	0.555	1.445	–	–	–	–	
25		0.969 6	0.548	1.392	3.931	1.804	6.058	0.565	1.435	–	–	–	–	

ANNEX B

(Foreword)

COMMITTEE COMPOSITION

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(*Continued on page* 15)

(*Continued from page* 14)

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